INTERFERENCE ESTIMATION AND MITIGATION FOR STAP USING THE TWO-DIMENSIONAL WOLD DECOMPOSITION PARAMETRIC MODEL

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ABSTRACT

We develop parametric modeling and estimation methods for STAP data based on the results of the 2-D Wold-like decomposition. We show that the same parametric model that results from the 2-D Wold-like orthogonal decomposition naturally arises as the physical model in the problem of space-time processing of airborne radar data. We exploit this correspondence to derive computationally efficient parametric fully adaptive and partially adaptive detection algorithms. Having estimated the parametric models of the noise and interference components of the field, the estimated parameters are substituted into the parametric expression of the covariance matrix to obtain an estimate of the interference-plus-noise covariance matrix. Hence the fully-adaptive weight vector is obtained. Moreover, it is proved that it is sufficient to estimate only the spectral support parameters of each interference component in order to obtain a projection matrix onto the subspace orthogonal to the interference subspace. The proposed partially adaptive parametric processing algorithm employs this property. The proposed parametric interference mitigation procedures can be applied even when only the information in a single range gate is available, thus achieving high performance gain when the data in the different range gates cannot be assumed stationary.

1. INTRODUCTION

We propose a new approach for parametric modeling and estimation of space-time airborne radar data, based on the 2-D Wold-like decomposition of random fields. The goal of space-time adaptive processing is to manipulate the available data to achieve high gain at the target angle and Doppler and maximal mitigation along both the jamming and clutter lines. Because the interference covariance matrix is unknown a priori, it is typically estimated using sample covariances obtained from averaging over a few range gates. Next, a weight vector is computed from the inverse of the sample covariance matrix, [1]-[5]. In [8], an approach that bypasses the need to estimate the covariance matrix was presented: The data collected in a single range gate was employed to obtain a least squares estimate of the signal power at each hypothesized DOA, through evaluation of a weight vector constrained to null the unknown interference and noise. In [9] a simple ad-hoc model of the clutter signal and covariance matrix is proposed. The model represents the spectral density of the clutter as a sum of Gaussian-shaped humps along the support of the clutter ridge. In [10] this model is employed to estimate the clutter covariance matrix from the data observed in a single range gate.

In this paper, we suggest to adopt the 2-D Wold-like decomposition of random fields, [6], as the parametric model of the observed data. Employing this model, we derive computationally efficient algorithms useful for parametrically estimating both the jamming and clutter fields. The estimation procedure we propose is capable of producing estimates of the interference signals parametric models even from the information in a single range gate. Hence, no averaging over a few range gates is required, achieving high performance gain in the practical case when the data in the different range gates is non-stationary. Having estimated the interference terms parametric models, their covariance matrix can be evaluated based on the estimated parameters. Moreover, the problem of evaluating the rank of the low-rank covariance matrix of the interference is solved as a by-product of obtaining the parametric estimates of the interference components. Once the parametric models of the interference components have been estimated, several alternative detection procedures are available. In this paper we present two such methods: the parametric fully-adaptive processing, and the parametric partially-adaptive processing.

2. THE RANDOM FIELD MODEL

In this section we shall briefly describe the 2-D Wold-like decomposition of random fields, [6]. Let $\{y(n,m)\}$, $(n,m) \in \mathcal{Z}^2$, be a complex valued, regular, homogeneous random field. Then, $y(n,m)$ can be uniquely represented by the orthogonal decomposition

$$y(n,m) = w(n,m) + v(n,m) \ . \qquad (1)$$

The field $\{v(n,m)\}$ is a deterministic random field. The field $\{w(n,m)\}$ is purely-indeterministic and has a unique

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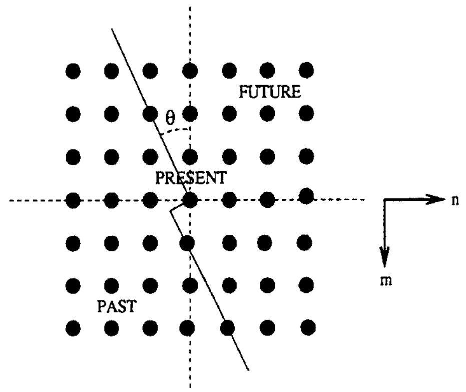

Figure 1: RNSHP support; example with $\alpha = 2$ and $\beta = 1$.

white innovations driven moving average representation, given by

$$w(n,m) = \sum_{(0,0)\preceq(k,\ell)} b(k,\ell)u(n-k,m-\ell) \qquad (2)$$

where $\sum_{(0,0)\preceq(k,\ell)} b^2(k,\ell) < \infty$; $b(0,0) = 1$, and $\{u(n,m)\}$ is the innovations field of $\{y(n,m)\}$. The notation $\preceq$ implies that the summation is performed over all the samples that are in the "past" of the (n,m) sample, where the past is defined with respect to any selected choice of NSHP total-ordering on the 2-D lattice. (See, for example, Fig. 1.) It is possible to define, [6], a family of NSHP total-order definitions such that the boundary line of the NSHP has a rational slope. Let α and β be two coprime integers, such that $\alpha \neq 0$. The angle θ of the slope is given by $\tan\,\theta = \beta/\alpha$. (See, for example, Fig. 1.) A NSHP of this type is called *rational non-symmetrical half-plane* (RNSHP). For the case where $\alpha = 0$ the RNSHP is uniquely defined by setting $\beta = 1$. (For the case where $\beta = 0$ the RNSHP is uniquely defined by setting $\alpha = 1$.) We denote by O the set of all possible RNSHP definitions on the 2-D lattice, (*i.e.*, the set of all NSHP definitions in which the boundary line of the NSHP has a rational slope). The introduction of the family of RNSHP total-ordering definitions results in the following countably infinite orthogonal decomposition of the deterministic component of the random field:

$$v(n,m) = p(n,m) + \sum_{(\alpha,\beta)\in O} e_{(\alpha,\beta)}(n,m)\ . \qquad (3)$$

The random field $\{p(n,m)\}$ is called *half-plane deterministic*. The field $\{e_{(\alpha,\beta)}(n,m)\}$ is the *evanescent* component that corresponds to the RNSHP total-ordering definition $(\alpha,\beta) \in O$.

Hence, if $\{y(n,m)\}$ is a 2-D regular and homogeneous random field, then $y(n,m)$ can be uniquely represented by the orthogonal decomposition

$$y(n,m) = w(n,m) + p(n,m) + \sum_{(\alpha,\beta)\in O} e_{(\alpha,\beta)}(n,m)\ . \quad (4)$$

It is further shown in [6] that the spectral measures of the decomposition components in (4) are mutually singular. A model for the evanescent field which corresponds to the RNSHP defined by $(\alpha,\beta) \in O$ is given by

$$\begin{aligned} e_{(\alpha,\beta)}(n,m) &= \sum_{i=1}^{I^{(\alpha,\beta)}} e_i^{(\alpha,\beta)}(n,m) \qquad (5) \\ &= \sum_{i=1}^{I^{(\alpha,\beta)}} s_i^{(\alpha,\beta)}(n\alpha - m\beta)\exp\!\left(j2\pi\frac{\nu_i^{(\alpha,\beta)}}{\alpha^2+\beta^2}(n\beta + m\alpha)\right) \end{aligned}$$

where the 1-D purely-indeterministic, complex valued processes $\{s_i^{(\alpha,\beta)}(n\alpha - m\beta)\}$ and $\{s_j^{(\alpha,\beta)}(n\alpha - m\beta)\}$, are zero-mean and mutually orthogonal for all $i \neq j$. Hence, the "spectral density function" of each evanescent field has the form of a countable sum of 1-D delta functions which are supported on lines of rational slope in the 2-D spectral domain.

One of the half-plane-deterministic field components, which is of prime importance in the STAP problem is the harmonic random field

$$h(n,m) = \sum_{p=1}^{P} C_p \exp\left(j2\pi(n\omega_p + m\nu_p)\right) \qquad (6)$$

where the C_p's are mutually orthogonal random variables, and (ω_p, ν_p) are the spatial frequencies of the pth harmonic.

3. THE STAP MODEL AND THE 2-D WOLD DECOMPOSITION

The random field parametric model that results from the 2-D Wold-like orthogonal decomposition naturally arises as the physical model in the problem of space-time processing of airborne radar data. In the latter problem the target signal is modeled as a random amplitude complex exponential where the exponential is defined by a space-time steering vector that has the target's angle and Doppler. In other words, in the space-time domain the target model is that of a 2-D harmonic component similar to (6). The purely-indeterministic component of the space-time field is the sum of a white noise field due to the internally generated receiver amplifier noise, and a colored noise field due to the sky noise contribution. The presence of a jammer results in a barrage of noise localized in angle and distributed over all Doppler frequencies. Thus, in the angle-Doppler domain each jammer contributes a 1-D delta function located at a specific angle, and therefore parallel to the Doppler axis. In the space-time domain each jammer is modeled as an evanescent component with $(\alpha,\beta) = (1,0)$ such that its 1-D modulating process is a white noise process. The ground clutter results in an additional evanescent component of the observed 2-D space-time field. The clutter echo from a single ground patch has a Doppler frequency that linearly depends on its aspect with respect to the platform. Hence, clutter from all angles lies in a "clutter ridge", supported on a diagonal line (that generally wraps around in Doppler) in the angle-Doppler domain. A model of the clutter field is then given by (5) with (α,β) such that $\tan\beta/\alpha$ corresponds to the slope of the clutter ridge. Since the rational numbers

are dense in the set of real numbers, an irrational slope of the clutter ridge can be approximated arbitrarily close, by a rational one. Hence any clutter signal can be either exactly modeled, or approximated by an evanescent field.

We therefore conclude that the foregoing derivation opens the way for new *parametric* solutions that can simplify and improve existing methods of STAP.

4. ESTIMATION OF THE COMPONENTS PARAMETERS: PROBLEM DEFINITION

We next state our assumptions and introduce some necessary notations. Let $\{y(n,m)\}$, $(n,m) \in D$ where $D = \{(i,j)|0 \le i \le S-1, 0 \le j \le T-1\}$ be the observed random field.

Assumption 1: The purely-indeterministic component $\{w(n,m)\}$ is a zero mean circular complex valued random field.

Assumption 2: The number $I = \sum_{(\alpha,\beta)\in O} I^{(\alpha,\beta)}$ of evanescent components in the field, is *a-priori* known. This assumption can be later relaxed.

Assumption 3: For each evanescent field $\{e_i^{(\alpha,\beta)}\}$, the modulating 1-D purely-indeterministic process $\{s_i^{(\alpha,\beta)}\}$ is a zero-mean circular complex valued process.

Let $\mathbf{y} = [y(0,0),\ldots,y(0,T-1),\ldots,y(S-1,T-1)]^T$, and let $\mathbf{w}$, $\mathbf{e}_i^{(\alpha,\beta)}$ be similarly defined. Let

$$\begin{aligned}&\boldsymbol{\xi}_i^{(\alpha,\beta)} = \\ &\Big[s_i^{(\alpha,\beta)}(0), s_i^{(\alpha,\beta)}(-\beta),\ldots,s_i^{(\alpha,\beta)}(-(T-1)\beta),\\ &s_i^{(\alpha,\beta)}(\alpha), s_i^{(\alpha,\beta)}(\alpha-\beta),\ldots,s_i^{(\alpha,\beta)}(\alpha-(T-1)\beta),\\ &\ldots,s_i^{(\alpha,\beta)}((S-1)\alpha),\ldots,s_i^{(\alpha,\beta)}((S-1)\alpha-(T-1)\beta)\Big]^T\end{aligned} \quad (7)$$

be the vector whose elements are the observed samples from the 1-D modulating process $\{s_i^{(\alpha,\beta)}\}$. Define

$$\begin{aligned}&\mathbf{v}^{(\alpha,\beta)} = \\ &[\,0,\ \alpha,\ \ldots,\ (T-1)\alpha,\\ &\beta,\ \beta+\alpha,\ \ldots,\ \beta+(T-1)\alpha,\ \ldots,\ldots,\\ &(S-1)\beta,\ (S-1)\beta+\alpha,\ \ldots,\ (S-1)\beta+(T-1)\alpha\,]^T\end{aligned} \quad (8)$$

Given a scalar function $f(v)$, we will denote the matrix, or column vector, consisting of the values of $f(v)$ evaluated for all the elements of $\mathbf{v}$, where $\mathbf{v}$ is a matrix, or a column vector, by $f(\mathbf{v})$. Using this notation, we define

$$\mathbf{d}_i^{(\alpha,\beta)} = \exp(j2\pi\frac{\nu_i^{(\alpha,\beta)}}{\alpha^2+\beta^2}\mathbf{v}^{(\alpha,\beta)}) . \quad (9)$$

Thus, using (5), we have that

$$\mathbf{e}_i^{(\alpha,\beta)} = \boldsymbol{\xi}_i^{(\alpha,\beta)} \odot \mathbf{d}_i^{(\alpha,\beta)} , \quad (10)$$

where $\odot$ denotes an element by element product of the vectors.

Note that whenever $n\alpha-m\beta = k\alpha-\ell\beta$ for some integers n,m,k,ℓ such that $0 \le n,k \le S-1$ and $0 \le m,\ell \le T-1$, the same element of $\boldsymbol{\xi}_i^{(\alpha,\beta)}$ appears more than once in the vector. It can be shown, [7], that for a rectangular observed field of dimensions $S \times T$ the number of *distinct* samples from the random process $\{s_i^{(\alpha,\beta)}\}$ that are found in the observed field is $N_c = (S-1)|\alpha|+(T-1)|\beta|+1-(|\alpha|-1)(|\beta|-1)$. This is because N_c is the number of different "columns" one can define on such a rectangular lattice for a RNSHP defined by (α,β). We therefore define the *concentrated version*, $\mathbf{s}_i^{(\alpha,\beta)}$ of $\boldsymbol{\xi}_i^{(\alpha,\beta)}$ to be an N_c dimensional column vector of non-repeating samples of the process $\{s_i^{(\alpha,\beta)}\}$. Thus for any (α,β) we have that

$$\boldsymbol{\xi}_i^{(\alpha,\beta)} = \mathbf{A}_i^{(\alpha,\beta)}\mathbf{s}_i^{(\alpha,\beta)} \quad (11)$$

where $\mathbf{A}_i^{(\alpha,\beta)}$ is rectangular matrix of zeros and ones which replicates rows of $\mathbf{s}_i^{(\alpha,\beta)}$.

Note however that due to boundary effects, the vector $\mathbf{s}_i^{(\alpha,\beta)}$ is not composed of consecutive samples from the process $\{s_i^{(\alpha,\beta)}\}$ unless $|\alpha| \le 1$ or $|\beta| \le 1$. In other words, for some arbitrary α and β there are missing samples in $\mathbf{s}_i^{(\alpha,\beta)}$. We note that the covariance matrix $\mathbf{R}_i^{(\alpha,\beta)}$ which characterizes the process $\{s_i^{(\alpha,\beta)}\}$ is defined in terms of the concentrated version vector $\mathbf{s}_i^{(\alpha,\beta)}$ *i.e.*, $\mathbf{R}_i^{(\alpha,\beta)} = E[\mathbf{s}_i^{(\alpha,\beta)}(\mathbf{s}_i^{(\alpha,\beta)})^H]$ and not in terms of the covariance matrix of the vector $\boldsymbol{\xi}_i^{(\alpha,\beta)}$, $\tilde{\mathbf{R}}_i^{(\alpha,\beta)} = E[\boldsymbol{\xi}_i^{(\alpha,\beta)}(\boldsymbol{\xi}_i^{(\alpha,\beta)})^H]$. The matrix $\tilde{\mathbf{R}}_i^{(\alpha,\beta)}$ is a singular matrix, given by $\tilde{\mathbf{R}}_i^{(\alpha,\beta)} = \mathbf{A}_i^{(\alpha,\beta)}\mathbf{R}_i^{(\alpha,\beta)}\left(\mathbf{A}_i^{(\alpha,\beta)}\right)^T$.

Since the evanescent components $\{e_i^{(\alpha,\beta)}\}$, are mutually orthogonal, and since all the evanescent components are orthogonal to the purely-indeterministic component, we conclude that $\mathbf{\Gamma}$, the covariance matrix of $\mathbf{y}$, has the form

$$\mathbf{\Gamma} = \mathbf{\Gamma}_{\mathrm{PI}} + \sum_{(\alpha,\beta)\in O}\sum_{i=1}^{I^{(\alpha,\beta)}} \mathbf{\Gamma}_i^{(\alpha,\beta)} , \quad (12)$$

where $\mathbf{\Gamma}_i^{(\alpha,\beta)}$ is the covariance matrix of $\mathbf{e}_i^{(\alpha,\beta)}$.

Using (10) and (5) we find that

$$\mathbf{\Gamma}_i^{(\alpha,\beta)} = \tilde{\mathbf{R}}_i^{(\alpha,\beta)} \odot (\mathbf{d}_i^{(\alpha,\beta)}(\mathbf{d}_i^{(\alpha,\beta)})^H) . \quad (13)$$

5. PARAMETRIC ESTIMATION OF THE INTERFERENCE COMPONENTS

In this section we derive a computationally efficient algorithm for estimating both the jamming and clutter fields, based on the above results. The proposed estimation algorithm of the spectral support parameters of the evanescent field, α,β and $\nu_i^{(\alpha,\beta)}$ is based on the observation (see the evanescent field model (5)) that for a fixed $c = n\alpha - m\beta$ (*i.e.*, along a line on the sampling grid), the samples of the evanescent component are the samples of a 1-D constant amplitude harmonic signal, whose frequency is $\nu_i^{(\alpha,\beta)}$. The algorithm is implemented by the following three-step procedure:

In the presence of an evanescent component, the peaks of the observed field periodogram are concentrated along a straight line, such that its slope is defined by the two coprime integers α and β. Hence, several alternative approaches for obtaining an initial estimate of the spectral

support parameters of the evanescent component can be derived by taking the Radon or Hough transforms, [12], of the observed field periodogram. (The current implementation employs the Hough transform for detecting straight lines in 2-D arrays). However, due to noise presence, this estimate may perturbate. Since on a finite dimension observed field only a finite number of possible (α, β) pairs may be defined, the output of the initial stage is a set of possible (α, β) pairs such that the ratio $\frac{\beta}{\alpha}$ is close to the ratio obtained for the (α, β) pair estimated by the Hough transform.

For each possible (α, β) pair we next evaluate the frequency parameter of the evanescent component, $\nu_i^{(\alpha,\beta)}$. Assuming the considered (α, β) pair is the correct one, we know that in the absence of background noise, for a fixed $c = n\alpha - m\beta$ (*i.e.*, along a line on the sampling grid), the samples of the evanescent component are the samples of a 1-D constant amplitude harmonic signal, whose frequency is $\nu_i^{(\alpha,\beta)}$. Hence, by considering the samples along such a line we obtain samples of a 1-D constant amplitude harmonic signal whose frequency $\nu_i^{(\alpha,\beta)}$ can be easily estimated using any standard frequency estimation algorithm (*e.g.*, the 1-D DFT).

The test for detecting the correct (α, β) and $\nu_i^{(\alpha,\beta)}$ is then based on multiplying the observed signal $y(n, m)$ by $\exp(-j2\pi \frac{\hat{\nu}_i^{(\hat{\alpha},\hat{\beta})}}{\hat{\alpha}^2+\hat{\beta}^2}(n\hat{\beta} + m\hat{\alpha}))$, for each of the considered α, β and $\nu_i^{(\alpha,\beta)}$ triplets, and evaluating the variance of this signal along a line on the sampling grid such that $c = n\alpha - m\beta$. Clearly, the best estimate of α, β and $\nu_i^{(\alpha,\beta)}$ is the one that results in minimal variance for the 1-D sequence, as in the absence of noise the correct α, β and $\nu_i^{(\alpha,\beta)}$ result in a zero variance.

Having estimated the spectral support parameters of each evanescent component, we take the approach of first estimating a *non-parametric* representation of its 1-D purely-indeterministic modulating process $\{s_i^{(\alpha,\beta)}\}$, and only at a second stage we estimate the parametric models of these processes. Hence, in the first stage we estimate the particular values which the vectors $\xi_i^{(\alpha,\beta)}$ take for the given realization, *i.e.*, we treat these as unknown constants. The estimation procedure is implemented as follows: Multiplying the observed signal $y(n, m)$ by $\exp(-j2\pi \frac{\hat{\nu}_i^{(\hat{\alpha},\hat{\beta})}}{\hat{\alpha}^2+\hat{\beta}^2}(n\hat{\beta} + m\hat{\alpha}))$ and evaluating the arithmetic mean of this signal along a line on the sampling grid such that $c = n\alpha - m\beta$, we have

$$\hat{s}_i^{(\alpha,\beta)}(c) = \frac{1}{N_s} \sum_{n\hat{\alpha}-m\hat{\beta}=c} y(n, m) \exp(-j2\pi \frac{\hat{\nu}_i^{(\alpha,\beta)}}{\hat{\alpha}^2+\hat{\beta}^2}(n\hat{\beta} + m\hat{\alpha})) \quad (14)$$

where N_s denotes the number of the observed field samples that satisfy the relation $n\alpha - m\beta = c$. Once we obtained the sequence of estimated samples from the 1-D modulating process $\{s_i^{(\alpha,\beta)}\}$, the problem of estimating its parametric model becomes entirely a 1-D estimation problem. Assuming the modulating process is an AR process, and applying to the sequence an AR estimation algorithm (see, *e.g.*, [13]) we obtain estimates of the modulating process parameters, as well.

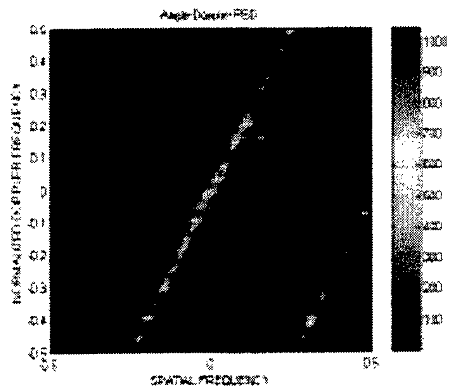

Figure 2: Spectral density of the observed field.

Finally, it is important to note that we solve the difficult problem of evaluating the rank of the low-rank covariance matrix of the interference as a by-product of obtaining the parametric estimates of the interference components: Denote the number of evanescent components (interference sources) of the field by Q. It is then shown in [11] that the rank of the interference covariance matrix is given by $\text{rank}(\mathbf{\Gamma}) = S\sum_{k=1}^{Q} |\alpha_k| + T\sum_{k=1}^{Q} |\beta_k| - \sum_{k=1}^{Q} |\alpha_k| \sum_{k=1}^{Q} |\beta_k|$. In fact the special case where $Q = 1$ and $\alpha = 1$ is the well known Brennan rule, [3], of the rank of the clutter covariance matrix.

6. PARAMETRIC FULLY ADAPTIVE PROCESSING

Having estimated the parametric models of the purely indeterministic and evanescent components of the field, the estimated parameters can be substituted into (12)-(13) to obtain an estimate of the interference-plus-noise covariance matrix $\mathbf{\Gamma}$.

Let $\mathbf{v}_t$ denote the target steering vector, given by

$$\mathbf{v}_t = \mathbf{b}(\varpi_t) \otimes \mathbf{a}(\vartheta_t) . \quad (15)$$

Assuming a linear, uniformly spaced, sensor array and a uniform CPI are employed in our model, the spatial steering vector $\mathbf{a}(\vartheta)$ and the temporal steering vector $\mathbf{b}(\varpi)$ are given by

$$\mathbf{a}(\vartheta) = [1, e^{j2\pi\vartheta}, \cdots, e^{j2\pi(S-1)\vartheta}]$$
$$\mathbf{b}(\varpi) = [1, e^{j2\pi\varpi}, \cdots, e^{j2\pi(T-1)\varpi}] ,$$

respectively. It is well known (*e.g.*, [3], p. 57) that the optimum space-time filter is given to within a scale factor by

$$\mathbf{w} = \mathbf{\Gamma}^{-1}\mathbf{v}_t , \quad (16)$$

The test statistic $z(\varpi, \vartheta)$ is then given by

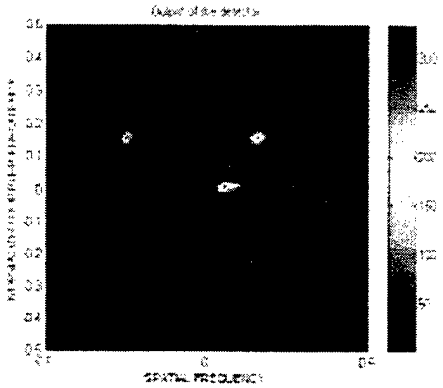

Figure 3: The test statistic $z(\varpi,\vartheta)$.

$$z(\varpi,\vartheta) = \mathbf{w}^H(\varpi,\vartheta)\mathbf{y} = \mathbf{v}_t^H(\varpi,\vartheta)(\mathbf{\Gamma}^{-1})^H\mathbf{y} . \quad (17)$$

Let $\chi_f = (\mathbf{\Gamma}^{-1})^H\mathbf{y}$. We thus have

$$z(\varpi,\vartheta) = \mathbf{v}_t^H(\varpi,\vartheta)\chi_f = \mathbf{b}^H(\varpi) \otimes \mathbf{a}^H(\vartheta)\chi_f . \quad (18)$$

Reorganizing the elements of χ_f into a $T \times S$ matrix Ψ where the elements of the kth row of Ψ are $\chi_f((k-1)S+1)\ldots\chi_f(kS)$, we conclude that for a linear, uniformly spaced, sensor array and uniform CPI the test statistic is given by

$$z(\varpi,\vartheta) = \sum_{p=1}^{T}\sum_{q=1}^{S} e^{-j2\pi(p-1)\varpi}e^{-j2\pi(q-1)\vartheta}\Psi(p,q) . \quad (19)$$

Thus, $z(\varpi,\vartheta)$ and $\Psi(p,q)$ are a 2-D DFT pair, and the test is equivalent to finding the 2-D frequency where the 2-D DFT of $\Psi(p,q)$ is maximal.

To illustrate the operation of the proposed solution we resort to numerical evaluation of some specific examples. Consider a 2-D observed random field consisting of a sum of a purely-indeterministic component (background noise), a single evanescent (interference) component, and three harmonic components (targets). The purely-indeterministic component is a complex valued circular Gaussian white noise field. The evanescent component spectral support parameters are $(\alpha,\beta) = (1,-2)$, $\nu^{(1,-2)} = 0$. The modulating 1-D purely indeterministic process of this evanescent component is a first order Gaussian AR process, such that its driving noise variance $(\sigma^{(1,-2)})^2 = 2$, and $a^{(1,-2)}(1) = -0.5$. There are three targets which are located at $(0.05, 0)$, $(0.15, 0.15)$ and $(-0.25, 0.15)$, respectively. The observed field dimensions are 48×48.

Let us define the experimental variance of each of the field components as $E_w = \mathbf{w}^H\mathbf{w}$ for the purely indeterministic component; $E_e = (\mathbf{e}^{(\alpha,\beta)})^H\mathbf{e}^{(\alpha,\beta)}$ for the evanescent component; and $E_{h_k} = \mathbf{h}_k^H\mathbf{h}_k$, $k = 1,2,3$, for each of the harmonic components, where $\mathbf{h}_k$ is defined in the same way $\mathbf{w}$ and $\mathbf{e}^{(\alpha,\beta)}$ are defined. In this example we have $\frac{E_e}{E_w} =$ 6dB, while for the three targets we have $\frac{E_{h_1}}{E_w} = -12.8$dB, $\frac{E_{h_2}}{E_w} = -14.5$dB, $\frac{E_{h_3}}{E_w} = -15$dB. Due to the strong interference component, the presence of the three targets is hard to detect in the observed data whose power spectral density is depicted in Fig. 2. However these targets are easily detected by the test statistic $z(\varpi,\vartheta)$ depicted in Fig. 3. In Fig. 3, $z(\varpi,\vartheta)$ is depicted as a function of the two-dimensional frequencies, *i.e.*, angle and Doppler.

7. PARAMETRIC PARTIALLY ADAPTIVE PROCESSING

Recall that

$$\mathbf{\Gamma}_i^{(\alpha,\beta)} = (\mathbf{A}_i^{(\alpha,\beta)}\mathbf{R}_i^{(\alpha,\beta)}(\mathbf{A}_i^{(\alpha,\beta)})^T) \odot (\mathbf{d}_i^{(\alpha,\beta)}(\mathbf{d}_i^{(\alpha,\beta)})^H) . \quad (20)$$

Having estimated α,β and $\nu_i^{(\alpha,\beta)}$ using the algorithm in Section 5, the vector $\mathbf{d}_i^{(\alpha,\beta)}$ is known. Hence, demodulating $\mathbf{e}_i^{(\alpha,\beta)}$, we conclude using (10) that the demodulated vector which we denote by $\bar{\mathbf{e}}_i^{(\alpha,\beta)}$ is given by

$$\bar{\mathbf{e}}_i^{(\alpha,\beta)} = \mathbf{e}_i^{(\alpha,\beta)} \odot ((\mathbf{d}_i^{(\alpha,\beta)})^H)^T . \quad (21)$$

From (11) we conclude that the covariance matrix of $\bar{\mathbf{e}}_i^{(\alpha,\beta)}$is given by

$$\bar{\mathbf{\Gamma}}_i^{(\alpha,\beta)} = \mathbf{A}_i^{(\alpha,\beta)}\mathbf{R}_i^{(\alpha,\beta)}(\mathbf{A}_i^{(\alpha,\beta)})^T . \quad (22)$$

In the following it is proved that since α and β are already known, an orthogonal projection matrix onto the low-rank subspace spanned by the evanescent field covariance matrix can be found *without* estimating the parametric model of the evanescent field 1-D modulating process, and hence without estimating $\mathbf{R}_i^{(\alpha,\beta)}$. Moreover this result enables us to avoid the need in both evaluating the field covariance matrix, and in employing a computationally intensive eigenanalysis to the estimated covariance matrix.

More specifically, let us construct the following orthogonal projection matrix

$$\mathbf{T}_i^{(\alpha,\beta)} = \mathbf{A}_i^{(\alpha,\beta)}\left((\mathbf{A}_i^{(\alpha,\beta)})^T\mathbf{A}_i^{(\alpha,\beta)}\right)^{-1}(\mathbf{A}_i^{(\alpha,\beta)})^T (23)$$

It is easily verified (by substitution) that $\mathbf{T}_i^{(\alpha,\beta)}$ is an orthogonal projection onto the range space of $\bar{\mathbf{\Gamma}}_i^{(\alpha,\beta)}$ since for any ST dimensional vector $\mathbf{v}$

$$\bar{\mathbf{\Gamma}}_i^{(\alpha,\beta)}\mathbf{v} = \bar{\mathbf{\Gamma}}_i^{(\alpha,\beta)}\mathbf{T}_i^{(\alpha,\beta)}\mathbf{v} . \quad (24)$$

Also, $(\mathbf{T}_i^{(\alpha,\beta)})^2 = \mathbf{T}_i^{(\alpha,\beta)}$, and $(\mathbf{T}_i^{(\alpha,\beta)})^T = \mathbf{T}_i^{(\alpha,\beta)}$.

Note that since $\mathbf{A}_i^{(\alpha,\beta)}$ is a sparse matrix of zeros and ones *only*, the computation of $\mathbf{T}_i^{(\alpha,\beta)}$ is very simple.

The projection matrix onto the subspace orthogonal to the interference space is therefore given by $(\mathbf{T}_i^{(\alpha,\beta)})^\perp = \mathbf{I} - \mathbf{T}_i^{(\alpha,\beta)}$. Hence by projecting the demodulated observed data vector $\bar{\mathbf{y}} = \mathbf{y} \odot ((\mathbf{d}_i^{(\alpha,\beta)})^H)^T$ onto the subspace orthogonal to the interference subspace, a reduced dimension data vector given by $\tilde{\mathbf{y}} = \left((\mathbf{T}_i^{(\alpha,\beta)})^\perp\right)^H \bar{\mathbf{y}}$ is obtained, such that

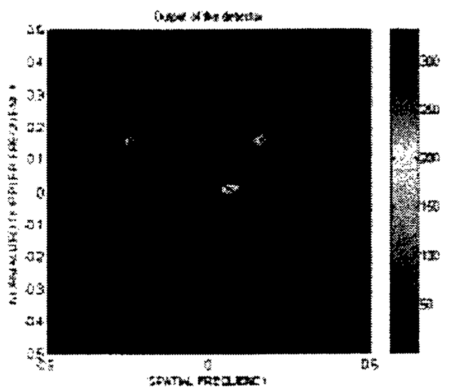

Figure 4: Spectral density of the field after being projected onto the subspace orthogonal to the interference subspace.

the interference contribution to the observed signal is mitigated. Remodulating $\tilde{\mathbf{y}}$ by evaluating $\tilde{\mathbf{y}} \odot \mathbf{d}_i^{(\alpha,\beta)}$, followed by sequentially applying this procedure to mitigate each of the interference sources, the detection problem is reduced to that of detecting a target in the presence of background noise only. Thus, in the special case where the background noise is known to be a white noise field, the statistical test is equivalent to finding the 2-D frequency where the 2-D DFT of the processed data vector (organized back into a 2-D array) is maximal.

As an example consider the same field as in the previous section. Due to the strong interference component, the presence of the three targets is hard to detect in the observed data whose power spectral density is depicted in Fig. 2. However these targets are easily detected in the processed data as illustrated in Fig. 4. This result is obtained without estimating the parametric model of the evanescent field 1-D modulating process, and hence without estimating the interference-plus-noise covariance matrix. Since both the estimation of the interference-plus-noise covariance matrix, as well as its analysis are saved, the proposed parametric partially adaptive processing method is robust and computationally attractive.

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